Septimiu Crivei

BASIC LINEAR ALGEBRA

Presa Universitară Clujeană

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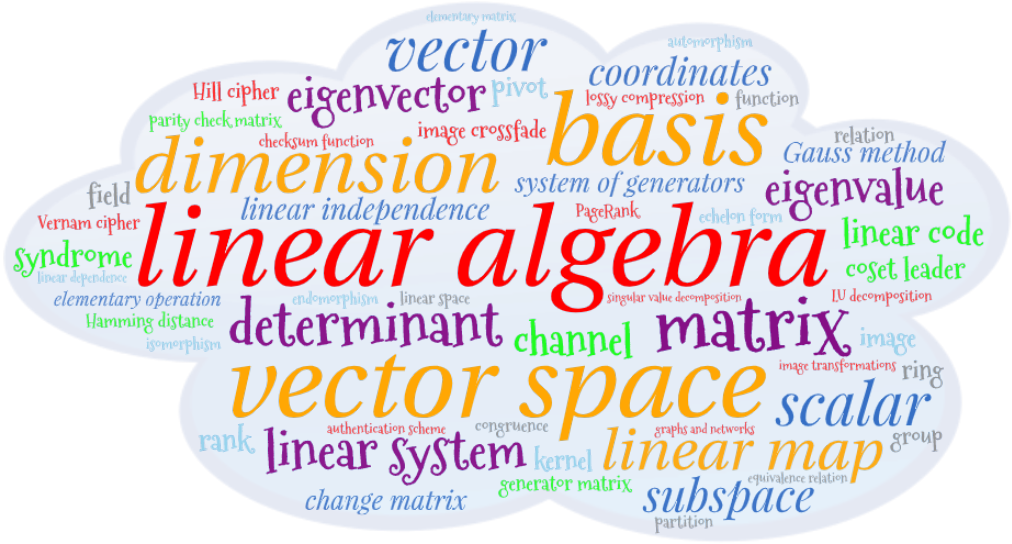

Presa Universitară Clujeană

2022

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ISBN 978-606-37-1582-2

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To my family

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Foreword

This textbook has grown up in the last twenty years, based on my courses of Algebra taught for first year students in Computer Science at the "Babeş-Bolyai" University of Cluj-Napoca. Besides the rather classical mathematical content, we also present some illustrations of the interplay between Algebra and Computer Science, which offers bidirectional applications. If we only restrict to Linear Algebra for Computer Science, then we should mention applications to several important research topics in Computer Science, such as Networks, Theory of Computation, Mathematics of Computing, Information Systems, Security and Privacy, Computing Methodologies and Applied Computing (see the corresponding addendum for some further details). The material is directed towards first year students in Computer Science, but it may be useful to anyone interested in an introduction to Linear Algebra and its applications to Computer Science.

The textbook is structured in four chapters, namely: *1. Preliminaries*, *2. Vector spaces*, *3. Matrices and linear systems* and *4. Introduction to coding theory*. Each chapter contains some extra material with related applications, and ends with a quiz with true or false questions and some projects to implement, which may help the reader to deepen the new concepts.

The first chapter is of a preliminary nature, setting the scene with some auxiliary needed notions, such as relations, functions, equivalence relations and partitions, algebraic structures with one and two operations as well as determinants. The core of the book consists of Chapters 2 and 3, which present the basics of Linear Algebra. In Chapter 2 we introduce and study vector spaces as algebraic generalizations of the vectors met in Physics. We also investigate subspaces, linear maps, quotient vector spaces, linear independence of vectors, bases and dimension of vector spaces. Chapter 3 reveals the strong connection between the abstract theory of vector spaces and the more practical concepts of matrices and linear systems. We study elementary operations and their applications, matrices of lists of vectors and linear maps, change of bases, linear systems and the Gauss method as well as eigenvectors and eigenvalues and some of their applications. The final chapter gives an introduction to coding theory, as a concrete application of the previously studied topics of Linear Algebra. We discuss the coding theory problem, Hamming distance, code representations, generator and parity check matrices as well as error-correcting and decoding.

The main content of the textbook is complemented by a series of addenda, namely *Historical notes* (on the concepts of matrix, determinant, vector space, basis and dimension, linear map, linear system, eigenvalue and eigenvector as well as on some scientists), *Computer Science topics using Linear Algebra* (according to the 2012 *Association for Computing Machinery* Classification System), *English-Romanian selected notions dictionary*, *Index of extra material* and *Index*.

Acknowledgements

I am grateful to Professors Iuliu Crivei and Gabriela Olteanu for inspiring conversations on the topics of this book and for their generous support.

I also thank my students throughout all these years, whose questions and interest have continuously motivated me to refine the taught material into its current form.

The author

Cluj–Napoca,
September 2022

ISBN: 978-606-37-1582-2